MATHEMATICAL FRAMEWORK FOR FINANCE AND INSURANCE

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Cipra, T. Financial and Insurance Formulas

Life is fundamentally risky, reflecting the pervasive out-of-equilibrium nature of the surrounding world. Risk is a companion to most of our daily activities, professional or private. Human society, with its technical development and population growth, introduces new kinds of risks: unemployment, strike, dysfunction of cities, ruptures of sensitive technological structures. Scientific and technical development and the growing interaction among the different organization levels of human society introduce an increasing complexity, leading often to an enhanced vulnerability. That is why the fields of finance and insurance become increasingly important and growing. However, this phenomena is accompanied both by scattering of information and difficult gaining of general overview in these fields.

Therefore, the overview of financial and insurance formulas seems to be a suitable aid, providing some mathematical framework. According to author’s own intention, this survey has several aims. First, the applicability of formulas in practice is stressed. Second, the formulas presented should be error-free. Third, the formulas should be systematically sorted and described including a simple denotation enabling a quick and operative search. Further, the formulas should be presented in the most frequent form occurring in practice. Finally, the formulas should be self-contained. For that reason, formulas of related fields (statistics, probability, demography, etc.) are also presented in final chapters.

The book consists of three fundamental parts divided further into individual chapters. As a rule, each chapter begins with the statement of key notions and concludes with recommendation for further reading. It should be stressed that the reader finds here not only formulas, but also corresponding methods, procedures and algorithms.

The first part is devoted to financial formulas and contains the following chapters:

- Simple Interest and Discount (the computation of simple interest, various forms of expressing time and interest rate, calendar conventions used in practice for the counting of difference between two dates, simple interest with principals credited \( m \)thly, simple discount).
- Compound Interest and Discount (the computation of compound interest and compound discount, the case of compound interest and discount convertible \( m \)thly, combination of simple and compound interest).
Continuous Interest and Discount (the transfer from the usual compounding interest model for the number of conversion periods approaching to infinity, the notion force of interest).

Classical Analysis of Interest Rates (risk free interest rate and real interest rate, term structure of interest rates).

Systems of Cash Flows (computation of present and future value, investment decision with a view to profitability of investments, internal rate of return, payback period, duration, different forms of convexity).

Annuities (classification of annuity types, annuity calculus, dynamic annuities, annuities payable mthly and continuously, debt amortization).

Depreciation (method of allocation of purchase cost of a depreciable asset across time, different kind of depreciation methods: straight line, arithmetical declining balance, sum of years digit, geometrical declining balance, arithmetical progressive balance).

Financial Instruments (discount securities, bonds, stocks, currencies).

Derivative Securities (general classification, forwards, futures, swaps, options).

Utility Theory (general notions — utility, utility function and examples in portfolio theory and cumulated wealth, measures of risk aversion).

Rate of Return and Financial Risk (rate of return, financial risk, VaR-value at risk, CaR-credit AT risk).

Portfolio Analysis and CAPM model (portfolio construction, portfolio with a risk-free asset, CAPM model).

Arbitrage Theory (basic notions: arbitrage opportunity, arbitrage-free principle, arbitrage pricing model).


The second part deals with insurance formulas and contains the following chapters:

Insurance Classification (classification of insurance risks, classification of insurance from the points of law and insurable interest, classification of life insurance from the points of establishment of technical provisions and profit benefits, classification of non-life insurance, classification of pension insurance).

Actuarial Demography (selected population indicators, life tables, mortality and survival modelling, multiple decrement models, multiple life functions, commutation functions).

Classical Life Insurance (basic concepts, symbols and calculation principles, technical provisions in life insurance, pure endowments, whole life and term insurance, further products of capital life insurance, life annuities, multiple life insurance, premium reserve and its implications, medical underwriting).

Modern Approaches to Life Insurance (critical illness insurance, flexible products of life insurance, unit linked, profit testing, embedded value, fair value).

Pension Insurance (basic concepts, defined benefit plan, defined contribution plan).

Classical Non-Life Insurance (basic concepts, premium calculations in non-life insurance, forms of non-life insurance and deductibles, technical provisions in non-life insurance, bonus-malus systems).

Risk Theory in Insurance (collective risk model, aggregate claim distribution, copula, credibility premium, ruin probability, deductible, calculations for bonus-malus system).

Health Insurance (classification according to type of insurance claims, classification of health insurer’s costs of claims, method of average costs, commutation functions in health insurance).

Reinsurance (basic concepts, types of reinsurance, solvency, alternative risk transfer).
The third part is concerned with formulas of related disciplines:

**Mathematical Compendium** (basic mathematical skills: powers with integral exponents, roots of real numbers, powers with rational exponents, powers with real exponents, logarithms, factorial and binomial coefficients, binomial theorem, sums of powers of natural numbers, numerical series, means, beta and gamma function).

**Probability Theory** (random events and probability, conditional probability and independent events, random variables and their basic characteristics, important discrete and continuous distributions, random vectors and their basic characteristics, transformation of random variables, conditional mean value, martingales, generating function, convolutions and sums of random variables, random sums of random variables, some inequalities, limit theorems of probability theory).

**Descriptive and Mathematical Statistics** (sampling theory: simple and stratified random samples, elementary statistical treatment, sample quantiles, measures of sample variability and sample concentration, measures of sample dependence, point and interval estimators, hypothesis testing, regression analysis, analysis of variance, multivariate statistical analysis).

**Econometrics** (multicollinearity, a priori restrictions, qualitative regressors, probit and logit models, random regressors and instrumental variable estimation, simultaneous equation models and 2SLS-estimator).

**Index Numbers** (indices as instruments of comparison, indices in practice, stock exchange indicators).

**Stochastic Processes** (classification and basic characteristics, Markov chains, Markov processes, important stochastic processes, spectral properties of stochastic processes).

**Statistical Analysis of Time Series** (predictions in time series, decomposition of time series, estimation of correlation and spectral characteristics, linear time series, nonlinear and financial time series, multivariate time series, Kalman filter).

Professor Cipra is well-known namely for the series of excellent books just from the fields of finance and insurance. Undoubtedly, the book reviewed belongs to this class, and, in the appropriate manner, it is complementary and unifying with respect to his preceding works. Last but not least, I must point out careful and accurate typing of complicated mathematical expressions and nice graphic arrangement.

The book can be recommended to university-level students with mathematical and economic specialization, to PhD candidates and researchers in the fields of finance and insurance. To an equal extent, it can be useful for employees of corresponding departments in banks and insurance companies. Finally, the third part of this book can serve as a reliable guidance for further education in related fields.