AN ESTIMATION OF OUTPUT GAP IN ROMANIAN ECONOMY USING THE DSGE APPROACH

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Abstract:
In this paper I use an open economy DSGE model and estimate it for Romanian economy using Bayesian techniques. Based on estimation I derive a smoothed estimation of the output gap. I compare the results with those from standard procedures to estimate the output gap, the Hodrick Prescott filter, the production function and an unobserved components model. The results show that the DSGE approach can give a better picture of the output gap and it is more consistent with the dynamics of Romanian economy.

Keywords: output gap, DSGE models, Bayesian techniques, monetary policy.

JEL Classification: C51, E32, E52.

1. Introduction

The standard model for monetary policy is the so-called the Neo-Wicksellian model, see Woodford (2003) for a detailed presentation. This framework is an extension of the standard New Keynesian framework, as outlined, for example, in Gali (2008), that specifies natural rates, like the potential output, or the natural rate of interest. The “Wicksellian” feature comes from the historic fact that it was Wicksell who introduced in the analysis of monetary policy the concept of natural rate of interest. Under this framework, the analysis of output gap is even more important, as the monetary authority seeks to replicate the flexible prices equilibrium, in which the interest rate equals the natural interest rate while the output gap is zero. Woodford (2003) also showed that the optimal monetary policy is the first best under flexible prices. Moreover, as Woodford (2003) pointed, since inflation dynamics depend on the model based output gap, the monetary policy should stabilize the output gap.

There is a long interest of economists in the concept of potential output or output gap as these concepts are essential in the analysis of business cycles and monetary policy. Initially, the researchers proposed univariate methods of extracting the otherwise unobservable potential output. Since this simple estimate-the-trend approach was from the very start simplistic, following approaches discussed the potential output in an economic founded system. One example is the Hodrick – Prescott filter that does not have a economic foundation, but also has some measuring problems (at tails).

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One solution that was proposed was the production function approach. Although more based on economic features, this solution lacked the link between inflation and the real side of the economy represented by the production function, as Hirose and Naganuma (2007) underlined.

A more complex and suitable approach proved to be the state space approach. Here the economist starts from the hypothesis that the potential output is unobservable and constructs a state space model that links state equations with observed equations. The estimation relies on the Kalman filter.

However, the soundest approach from both an economic and econometric point of view is the dynamic stochastic general equilibrium approach. It should be pointed that the DSGE based potential output should not be assessed against the univariate or multivariate filtered derived potential output, since they are very different concepts.

Several studies addressed the problem of estimating the output gap using the DSGE approach. For example, Basistha and Nelson (2003) used a New Keynesian model to estimate the output gap. Their approach was based on the definition of output gap in the New Keynesian framework. The estimation framework was based on the Kalman filter which was applied on a state space model.

For the case of Japan, Hirose and Naganuma (2007) estimated an output gap using an estimated DSGE model on US data. They stressed the advantages of a DSGE approach, however, they also advised for caution of such an approach, since the Bayesian approach is sensitive to the choice of priors. They suggested that choosing a certain monetary policy rule matters for the final results.

Bjornland, Leitemo and Maih (2008) extended Basistha and Nelson (2003) study by arguing that the estimated output gap should be consistent with the New Keynesian framework, and that inflation expectations should both rational and consistent with the theory. They also estimated the model for the US economy.

However, there is a very limited literature that addressed the problem of using the DSGE framework to compute the natural output in a small open economy, and practically no study for Romanian economy. Although some DSGE models were recently discussed for the case of Romania, like Caraiani (2008a) or Caraiani (2008b), their focus was on the estimation and analysis of the impulse response functions or on the monetary policy analysis. The purpose of this paper is to estimate an open economy DSGE model for Romania with Bayesian techniques and use the estimated model to compute the output gap. The results are analysed in the light of the dynamics of Romanian economy post 2000, especially in the light of the adoption of the inflation targeting regime (IT, hereafter), of the disinflationary process as well as of the accelerated growth period.

This paper is organized as follows. The next section introduces the model and explains its building blocks. I estimate the model using Bayesian techniques in the third section and I compare the results of the estimation with the reference literature. In the fourth section I estimate the output gap and analyse its dynamics in the light of the macro-dynamics in Romanian economy as well as comparing with results from standard procedures. I draw an assessment of the results and some possible extensions in the last section.
2. The Model

I use the model proposed by Beltran and Draper (2008) that investigated the problem of estimating the parameters of a small open economy. They extended the reference model in Monacelli (2003) by including habit formation. The model is characterized by two sectors: domestic producers and importers, and by sticky prices. It also introduces the computation of the output gap as the difference between the actual output and the flexible prices output.

There are four types of agents in the domestic block of the model. There are representative households, domestic producers, retailers and the monetary authority.

The domestic households maximize the expected lifetime utility. The utility function comprises consumption with habit formation and leisure. The solution to this problem results in a New Keynesian IS curve.

The domestic producers are monopolistic ones. In a typical way for New Keynesian (further as NK) models, the monopolistic producers face rigid prices. The representative firm maximizes the expected discounted value of the profits, under the constraint given by the demand curve and the monopolistic competition. Solving this problem leads to the New Keynesian Phillips curve for domestic producers.

The retailers also operate under Calvo price rigidity. Their optimization problem leads to a similar New Keynesian Phillips curve for importers.

The model is closed by adding the equations related to monetary policy, the market equilibrium, the real exchange rate, law of one price parity condition, the output gap and potential output equations, the foreign block, and by specifying the autoregressive processes for productivity shock.

In the next paragraphs, I present the model I use in the estimation and analysis from the next sections. The model is already in log-linear form:

\[ \pi_{h,t} = \beta E_t \pi_{h,t+1} + \frac{(1-\theta_H)(1-\beta\theta_H)}{\theta_H} mc_t + \varepsilon_{\pi_{h,t}} \]  
\[ \pi_{f,t} = \beta E_t \pi_{f,t+1} + \frac{(1-\theta_F)(1-\beta\theta_F)}{\theta_H} \psi_{\pi_{f,t}} + \varepsilon_{\pi_{f,t}} \]  
\[ \pi_t = \pi_{h,t} + \gamma \Delta x_t \]  
\[ mc = \phi y_t - (1+\phi)z_t + \gamma x_t + \sigma(1-h)^{-1}(c_t - hc_{t-1}) \]  
\[ q_t = (1-\gamma)x_t + \psi_{\pi_{f,t}} \]  
\[ \Delta\psi_{\pi_{f,t}} = \Delta x_t + \pi_{f,t}^* - \pi_{f,t} \]  
\[ \Delta x_t = \pi_{f,t} - \pi_{h,t} \]  
\[ (c - hc_{t-1}) = (y_t^* - hy_{t-1}^*) + \frac{1}{\sigma}(1-h)[(1-\gamma)x_t + \psi_{\pi_{f,t}}] \]  
\[ (i_t - E_t \pi_{t+1}) - (i_t^* - E_t \pi_{t+1}^*) = E_t[\Delta q_{t+1}] + \varepsilon_{q_{t+1}} \]
Equation (1) and (2) express the domestic inflation $\pi_{H,t}$ and imported inflation $\pi_{F,t}$ as typical New Keynesian curves. The domestic inflation $\pi_{H,t}$ depends on expected domestic inflation $\pi_{H,t+1}$ and marginal cost, $mc_t$. The coefficient $\theta_H$ expresses the Calvo rigidity with respect to domestic inflation $\epsilon_{\pi_H}$. $\epsilon_{\pi_F}$ is the domestic inflation shock.

In a similar way, the imported inflation $\pi_{F,t}$ depends on its own expected values as well as on $\psi_F$, the law of one price gap, with $\theta_F$ the parameter of price rigidity for imported inflation. Here $\epsilon_{\pi_F}$ is the imported inflation shock.

CPI inflation $\pi_t$ is determined in equation (3) as the sum of domestic inflation and the change in the terms of trade $x_t$. The coefficient $\gamma$ stands for the share of foreign produced goods in the consumption bundle. Terms of trade $x_t$ are given by the difference in the domestic inflation and imported inflation, equation (7).

Marginal cost $mc_t$ depends on the variables output $y_t$, productivity process $z_t$, terms of trade $x_t$ and on consumption $c_t$, equation (4). The parameter $\sigma$ is the coefficient of relative risk aversion, while $\phi$ is the inverse elasticity of labor supply. The coefficient $h$ characterizes the degree of habit formation.

Equation (5) shows the real exchange rate $q_t$ depending on terms of trade $x_t$ and law of one price gap, $\psi_{F,t}$. The law of one price gap is given in equation (6) as depending on changes in the nominal exchange rate $s_t$ and the difference between foreign inflation $\pi_{t,*}$ and imported inflation $\pi_{F,*}$.

Equation (8) is the consumption function characterized by habit formation, with $h$ the habit parameter. Consumption $c_t$ depends on foreign output $y_{t,*}$, terms of trade $x_t$ and law of one price gap $\psi_{F,t}$. The uncovered interest rate equation which links the real interest rate difference between domestic and foreign economies to the changes in the exchange rate. The variable $\epsilon_{q_t}$ is the shock on the real exchange rate.

Equation (10) shows the market clearing condition for a small open economy. The coefficient $\eta$ is the elasticity of substitution between domestic and foreign goods.

The monetary policy rule is given in equation (11) and is a Taylor type one. Besides
an interest smoothing coefficient, $\rho_r$, the equation also is characterized by the inflation coefficient, $\gamma_\pi$, and the output gap coefficient, $\gamma_x$. The shock on the interest rate is given by $\varepsilon_{rt}$.

Equations (12) to (14) express the output gap $\tilde{y}_t$, flexible prices output $y_{t}^{\text{flex}}$ and equilibrium terms of trade $y_{t}^{\text{flex}}$. As mentioned above, in the NK framework, the potential output is that level of output reached in the absence of nominal rigidities. In other words, the potential output is given by the flexible price output level. The natural output is determined by the real terms productivity process $z_t$ and equilibrium terms of trade $x_{t}^{\text{flex}}$. The equilibrium terms of trade is modelled as a combined process of its own lagged values $x_{t-1}^{\text{flex}}$ and of the productivity process $z_t$. Here the coefficient $A$ and $B$ are given by:

$$A = \frac{h\sigma \left[ \phi \gamma \eta (2 - \gamma) + 1 \right]}{\sigma \left[ \phi \gamma \eta (2 - \gamma) + 1 \right] + (1 - h)(1 - \gamma)^2 \phi}$$

$$B = \frac{\sigma (1 + \phi)}{\sigma \left[ \phi \gamma \eta (2 - \gamma) + 1 \right] + (1 - h)(1 - \gamma)^2 \phi}$$

The productivity process $z_t$ is modelled in equation (15), with $\rho_z$ the autocorrelation coefficient, and $\varepsilon_{zt}$ the productivity shock.

Finally, the foreign economy block is specified in equations (16) to (18) as a VAR model in $y_{t}^{*}$, $\pi_{t}^{*}$ and $r_{t}^{*}$, with $\omega_{ij}$ the coefficients of the VAR model. $\varepsilon_{y,t}^{*}$, $\varepsilon_{\pi,t}^{*}$ and $\varepsilon_{r,t}^{*}$ are the shocks on foreign variables.

### 3. Data and Estimation of the Model

I estimate the model given in the equations (1)-(18) using Bayesian techniques. The estimation was done for the period between 2000Q3 and 2008Q4 using quarterly data. I used as observed variables the domestic quarterly GDP, domestic inflation rate, domestic nominal interest rate, the real exchange rate as well as the euro area quarterly GDP and interest rate. Quarterly GDP was seasonally adjusted and expressed as a growth rate, where GDP is in constant prices at the level of year 2000. Quarterly inflation is the annualized GDP deflator. The quarterly interest rate is the average of the monthly interest rate during the current quarter. Both inflation and interest rate were de-trended given the disinflationary trend of Romanian inflation and interest rate after the year 2000.

The prior distributions were set following basically the paper of Beltran and Draper (2007) and previous results from Caraiani (2008a) or Caraiani (2008b). The foreign economy block was considered as a VAR model with the priors reflecting the expected sign of the parameters.

The estimation is based on two Metropolis-Hastings chains each one of 200,000 extractions. The multivariate statistics Brooks and Gelman (1998), Annex 4, indicate that convergence was achieved. The prior and posterior distributions are presented below in Table 1.
The estimated coefficients of price rigidity for both domestic inflation and imported inflation confirm the previous findings for Romanian economy, see Caraiani (2008a) or Caraiani (2008b): the firms adjust their prices, in average, every four quarters.

For the Taylor rule, the results indicate a low smoothing process of interest rate, similar to that in Caraiani (2008b), indicating that the central bank was not at all gradual in implementing the monetary policy. For inflation I obtained an expected value. As the national Bank adopted the inflation targeting regime, the inflation coefficient is high, estimated at 1.29. The national bank, as the estimates show, followed, first of all, the price stability. The coefficient associated to the output gap is higher than the usual findings in the literature suggesting that the national bank paid close attention
to the output fluctuations. The estimates for inflation coefficient are similar to those in previous estimation for Romania. At the same time, the coefficient for inflation and output gap are close to those obtained by Beltran and Draper (2008) for the informative priors.

The estimated coefficients of the VAR model of foreign economy have expected signs. The interest rate has a negative impact on output, $\omega_{13}$, while the impact on inflation is uncertain $\omega_{23}$ (inflation might react in a delayed way). An increase in inflation leads to higher output, $\omega_{33}$, while a positive shock in output leads to both higher inflation, $\omega_{21}$, and higher interest rate. A higher inflation implies that the central bank reacts by raising the interest rate, $\omega_{32}$. The positive shocks in a variable have positive effects on its own values, $\omega_{11}$, $\omega_{22}$, and $\omega_{33}$. Compared to the results in Beltran and Draper (2008), the coefficients have the expected sign, except for those for interest impact on inflation $\omega_{23}$, with some differences with respect to magnitudes.

The standard error of the shocks, $\sigma_{\pi_1}$, $\sigma_{\pi_2}$, $\sigma_q$, $\sigma_r$, $\sigma_z$, $\sigma_{\pi^*}$, $\sigma_{r^*}$ and $\sigma_{y^*}$ are estimated using the standard prior distribution in the literature, namely an inverted gamma distribution. The results show that the biggest variation is in the exchange rate, interest rate and domestic inflation, which is actually confirmed by the real data volatility of these macroeconomic time series.

4. An Analysis of the Output Gap in Romanian Economy

Most of the previous studies for Romania tried to estimate the output gap under standard econometric methods, like univariate filters, calibrated production functions, multivariate filters or SVAR models, like Stănică (2005), Scutaru and Stănică (2005), Dobrescu (2006), or Gălățescu et al. (2007).

Scutaru and Stănică (2005) used a structural VAR which they estimated using quarterly data between 1995 and 2003. They found that the inflationary shocks and technological shocks do matter for the output gap dynamics. The estimated output gap fluctuates between positive and negative signs.

Stănică (2005) used univariate methods which were basically variations of the local trend model to estimate the potential output between 1994 and 2003. He found a fluctuating pattern for the output gap that switches between negative and positive signs.

In an original contribution, Dobrescu (2006) argued that the estimated potential output should be based not only on domestic variables, like inflation, unemployment rate or wage, but also on foreign variable. Thus, the potential output is that output which is consistent with stable inflation, and sustainable foreign trade balance. In his view, the output gap reflects exclusively the demand pressure. He computed the output gap on quarterly data between 1991 and 2002 and found alternating signs for the output gap.

Gălățescu et al. (2007) used several methods to estimate the growth rate of potential GDP. The methods used were the production function, the HP filter, unobservable components models, and SVAR models. They argued that the natural GDP growth rate increased from 3 to 4% in 2001-2002 to 6% during the last years.

I discuss now the output gap derived from the estimated DSGE model for Romania. After estimating the linearized model in equations (1) to (18), I estimated the potential output and the output gap using the Kalman smoother. Figure 1 shows the estimated output gap.
In order to compare the results with those from other methods, I also derived an output gap measure using three of the most known standard procedures to measure the output gap: the Hodrick Prescott filter, an unobserved components model as well as the production function, see Annex 1, Annex 2 and Annex 3 for the results.

The approach proposed here has certain advantages over the previous methods used to derive output gap in Romania: it uses an open economy model, it is founded on a structural model with optimizing agents and it relies on the Bayesian estimation.

**Figure 1**

*Estimated Output Gap from New Keynesian Model*

The output gap is negative for two periods. First of all, during 2001Q2-2006Q1, the actual output is lower than the potential output. My explanation is based on two reasons: first of all, this period was a period of disinflation in which the main purpose of policy makers in Romania was to reduce the double digit inflation to a single digit; second of all, it took several years of growth until Romania’s growth accelerated. The second period of negative output gap started with the fourth quarter of 2008, and it is a reasonable assumption that the negative output gap will persist until the ongoing crisis ends.

There are also two periods of positive output gap. The first one is at the beginning of the period, between 2000Q3 and 2001Q1, when Romanian economy started to grow. The second period of growth started in 2006Q2, when Romania enjoyed an accelerated economic growth rate. We can see that the output gap increased toward the end of the year 2008, which seems reasonable in the light of the accelerated growth of the last years.

Comparing to the other methods applied to derive the output gap, the NK approach is the only one to produce positive output gaps starting with 2006, while the other detect in a wrong way a negative output gap for a few quarters in 2007. The other approaches also detect similar a negative output gap along the disinflation period, except for the year 2004 and one-two quarters in 2002.
The results here contradict the view proposed by previous studies of an output gap with alternating signs. For the period between 2001 and 2004, which can be also found in Stănică (2005) or Stănică and Stănică (2005), this study found a negative output gap which is probably related the disinflationary policy of the national bank.

The sequence of a period of negative output gap followed by a period of positive output gap after 2000 is supported by similar findings for some other CEE economies. Tetsuya (2008) used the production function approach as well as the multivariate Kalman filter for annual data between 2000 and 2008 for Slovakia. Based on the production function he found negative output gaps between 2001 and 2006, followed by positive output gaps from 2007. The multivariate Kalman filter resulted in a negative output gap between 2003 and 2006, and a positive output gap starting with 2007. However, Tetsuya (2008) showed that the estimated output gap using the multivariate Kalman filter is more consistent with the economic realities of Slovak economy.

We can also note the results in Benes et al. (2005) who used the projection model at Czech National Bank and estimated the output gap using the Kalman filter. They found a negative output gap for Czech Republic between 1998 and 2004.

5. Conclusion

The output gap is an essential concept in the theory and implementation of monetary policy. Such a concept is even more significant for a former transition economy now in the path towards the integration into the euro area. The difficulty comes from the fact that the output gap is an unobserved variable. Several methods were proposed in time to address this problem, but each one has some deficiencies.

In this paper, I estimated an open economy NK model for Romanian economy, and used the estimated model to compute the unobserved output gap. I compared this estimation with measures of output gaps derived from standard models. The NK based estimation of the output gap is more consistent with the dynamics of Romanian economy and, at the same time, is more founded from a theoretical point of view.

A sounder approach to the estimation of the output gap for a small open economy like Romania would be to add financial and monetary variables like credits, financial rigidities, etc.

References

Annex 1.

Computing the Output Gap Using Hodrick Prescott Filter

While it was created during the emergence of the real business cycles school and since then it became one of the most used tools in the business cycles analysis, the Hodrick and Prescott (1980) filter became in time one of the most used methods to filter macroeconomic time series.

The HP filter is supposed to extract in an optimal way the trend, which should be both stochastic and smooth and, at the same time, uncorrelated with the cyclical component. The HP filter implies the minimization of the following expression:

$$\min_{\text{trend}} \sum_{t} (y_t - \text{trend}_t)^2$$

where $y_t$ is the observed macroeconomic variable and $\text{trend}_t$ is the trend component estimated by the HP filter.
Here $T$ is the number of observations, $\lambda > 0$ is a parameter which penalizes the variation in the trend, $c_t$ is the cycle, and $x_t$ the trend component. As $\lambda$ grows, the degree of correction of cycles grows, while $x_t$ becomes smoother. Figure 2 shows the derived output gap using the HP filter using a value for $\lambda$ of 1600. A justification of the value for $\lambda$ can be found in Ravn and Uhlig (2002). The GDP series used is the same to be used throughout the paper, namely quarterly GDP in constant prices of the year 2000.

The HP filter was severely criticized from several perspectives. Two of the reference studies that revealed the weaknesses of the HP filter are Harvey and Jaeger (1993), and Cogley and Nason (1995).

Harvey and Jaeger (1993) criticized the HP filter because it induces spurious cycles. Moreover, the cyclical components obtained through this filter are distorted and thus they could lead to false conclusions regarding the short – run relations between the macroeconomic series.

Cogley and Nason (1995) analysed the effect of using the HP filter on trend-stationary and difference-stationary series. They showed that if one applies the HP filter to an integrated series, the HP filter can falsely generate business cycle dynamics that are not present in the real data. Thus, they ask for caution when analysing HP filtered data.
Annex 2.

Computing the Output Gap Using the Unobserved Components Model

A second approach to the computation of output gap is based on an unobserved components model. The model decomposes the observed series of seasonally adjusted GDP into its unobserved components: the trend, the cycle(s), and the irregular.

I use here a version of a model used in Stănică (2004), who used the local linear trend model, as given below:

\[ y_t = \mu_t + \psi_t + \epsilon_t \]  \hspace{1cm} (20)

\[ \mu_t = \mu_{t-1} + \beta_t \]  \hspace{1cm} (21)

\[ \beta_t = \beta_{t-1} + \xi_t \]  \hspace{1cm} (22)

The model decomposes the observed GDP series into a trend component \( \mu_t \) and a medium term cycle \( \psi_t \) of five years as well as an irregular component \( \epsilon_t \). The trend component is characterized by a fixed level and a stochastic slope \( \beta_t \).

The estimation is based on the Kalman filter that applies to the model written in state space form. The estimation indicates that there is strong convergence reached after 30 iterations.

Figure 3
Estimated Output Gap from Unobserved Components Model
Annex 3.

Computing the Output Gap Using the Production Function

The production function expresses the GDP as depending on the supply side factors: the inputs, employment $L_t$ and capital stock $K_t$, and an exogenous factor known as the Solow residual or total factor productivity, $A_t$.

$$Y_t = A_t (K_t)^{\alpha}(L_t)^{1-\alpha} \quad (23)$$

The coefficient $\alpha$ is the elasticity of capital with respect to capital, or the capital share. The production function here is assumed to be characterized by constant returns to scale.

The capital stock is constructed starting from the data about intangible assets in the economy at an annual level from which a capital stock in constant prices can be derived using the GDP deflator. Once an initial capital stock is derived for first quarter of 2000, the quarterly series is constructed using the following equation:

$$K_t = (1-\delta)K_{t-1} + I_t$$

where $\delta$ is the quarterly depreciation rate, while $I_t$ is the quarterly gross fixed capital formation in constant prices 2000. The parameter $\delta$ is set to 0.025 following the computations in Caraiani (2007).

The employment is the quarterly employment series, after seasonally adjustment. The total factor productivity series is derived as a residual from equation (23) using a value for the parameter $\alpha$ of 0.35 suggested by Caraiani (2007).

Finally, to express potential output, potential level of capital stock, employment and total factor productivity are used. For the capital stock, since a capacity utilization series is not available, and, at the same time, the sample period was characterized by growth, I simply assume that potential capital stock is equal to current capital stock. The potential total factor productivity is the HP filtered series of total factor productivity while the potential employment is the HP filtered actual employment. The results are shown in Figure 4.

Figure 4
Estimated Output Gap from Production Function

![Figure 4 Estimated Output Gap from Production Function](image-url)
Although the “production function” approach has certain advantages: like modelling the underlying factors that lead to changes in potential GDP, reflecting the supply side of the economy, there are also some limits to this approach.

The foremost limitation is the fact that this model is a very simplistic view of the economy. At the same time the model assume perfect competition on the factors market as well as homogenous factors. These features clearly do not characterize the CEE economies.

The capital stock series is also hard to compute for any economy, and clearly harder for a CEE economy that passed through turbulent restructurings. At the same time the parameters that are needed to estimate the potential output are the depreciation parameter and the capital share, which require stable and detailed data which is not always available for a CEE country.

Annex 4.

The Multivariate Converge Statistics Brook-Gelman