ECONOMIC AND SOCIAL HARMONIZATION 
OF SUSTAINABLE PUBLIC TRANSPORT

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Abstract:

The paper identifies, formulates and resolves several problems arising from dealing with the economic and social pillars of sustainable public transport. In the first part of the paper three pillars of sustainable transport are presented. The second part studies harmonization problems of economic, spatial and time accessibility of public transport and illustrates them by the Czech situation. The third part studies the cost-revenue-subsidy relation, emphasizes the right choice of fares and suggests a harmonization between the prices of the ticket for individual trip and the monthly one. The fourth part shows several important measures for getting the economic and social acceptability of public transport service, namely optimal choice of rolling stock structure, routes and frequencies, vehicle and crew scheduling.

Keywords: economic, social, ecological, sustainability, cost, revenue, subsidy, public transport, decision making, public administration

JEL Classification: R490, C650

1. Introduction

If mankind wants to survive in the long term, it must ensure its sustainable development. People must realize that they cannot indefinitely extend the scope of their claims.

Sustainable development, as an important global problem, has been widely examined and analysed mainly during the last three decades. In the late eighties, the environmental aspects were highlighted. Later, their social and economic impacts were considered, too. This is true in the decision making of public administration as well, e.g. as concerns quality improvement of recreation area (Vojáček and Pecáková, 2010) or public transport (Hensher, 2007).

We shall deal with the second example, i.e. with transport sustainability. Traffic accidents and congestion on the roads show the urgency of action in this direction. Banister (2008) has emphasized that a new, more flexible, sustainable mobility paradigm is necessary now.
Sustainable development of transport props on three pillars:

- **ecological** (keeping air pollutants, noise *etc.* endurable),
- **social** (attractiveness, accessibility, influence on social development),
- **economic** (acceptable costs, needs of subsidies).

We have to say that these three branches are not called “pillars” by all authors. Kennedy *et al.* (2005) call them “classic triangle of sustainability” and the notion of pillars they reserve for four essential components of the process of moving towards sustainable transportation, specifically: (1) the establishment of effective bodies for integrated land-use transportation planning; (2) the creation of fair, efficient and stable funding mechanisms; (3) strategic investments in major infrastructure; and (4) the support of investments through local design. Despite of it, we shall speak about ecological, social and economic pillars in the sequel.

It is obvious that enhancement of any two pillars causes deterioration of the remaining one: Less pollution and smaller costs (usually) cause worse accessibility and attractiveness of transport. On the other hand, less pollution and better accessibility causes growth of costs. And finally, better accessibility with smaller costs leads to worse ecological impact. Hence a **harmonization**, *i.e.* finding equilibrium among these three pillars, is a quite natural issue.

**Ecological pillar of public transport** has a specific position. Of course, any method of ecological impact improvement, which is effective in transport in general, applies to public transport as well. But, moreover, any growing of patronage share of public transport, with respect to individual car use, evokes reduction of pollution and other bad impacts of transport on the environment. In the other words, the more massive social pillar of public transport, the stronger ecological pillar. In accordance with that, the **most important harmonization questions concern the social and the economic pillars**. And it is necessary to add that in the period after the financial crisis the economic one is the last, but not the least at all.

In this paper, three types of harmonization are to be dealt:

- inside the social pillar - Section 2
- inside the economic pillar - Section 3
- between the economic and the social pillars – Section 4.

A harmonization problem can arise not only between the economic and social pillars, but also “inside” the social one. One can look for the equilibrium between the spatial and the time accessibility or among all three ones, including the economic accessibility.

We have to emphasize that, in our case, the word “equilibrium” means an optimal proportionality of two or three characteristic parameters.
2. Harmonization within the Social Pillar

The paper Geurs et al. (2009) describes a wide range of social impacts of transport. However, as concerns public transport, all these effects are strongly connected with its attractiveness with respect to its main substitute - individual car use. In the sequel, we shall deal with “direct” measures making public transport more attractive. Of course, we have to take into account that there exist “indirect” measures as well. They may make individual car use less attractive, see Fujii (2010). Maybe the most important ones inhere in road pricing and parking policy, as described e.g. in Jansson (2010).

Holmgren (2007) presented elasticity of public transport demand with respect to fares, supply vehicle kilometres, car ownership, price of petrol and income. Holmgren et al. (2008) analysed the decline in patronage and presented measures that could evoke a positive trend-break. One can see that there are inner factors inside the public transport system influencing its attractiveness, e.g. fares and supply vehicle kilometres. They can be unified into one embracive factor - the accessibility of public transport services for the passengers. We distinguish the following types of it:

- **Spatial accessibility**, representing, for each passenger, the sum of two distances: from the initial point to the nearest stop and from the target point to the nearest stop.
- **Time accessibility**, representing the time interval between the desired departure time and the factual one.
- **Economic accessibility**, representing the relation between the fare (= the price of the ticket) and the financial potentiality of the passenger.

Accessibility needs of elderly people are worthy of special consideration as presented in Xinyu et al. (2010).

Transport theorists and engineers often take into account passenger time loss as a main objective in their decision making on public transport, see e.g. Janáček, Linda, Ritschelová (2010). Therefore, they take care of time accessibility of the services. And, moreover, one has to emphasize that they seek to minimize the average time loss of passengers.

On the other hand, municipal and regional authorities like to postulate the necessity “to have the nearest stop within $d_{\text{max}}$ meters”. In the other words, they take care of space accessibility of the services. And, by contrast to the previous case, they intend to have the nearest stop within $d_{\text{max}}$ meters for any passenger, i.e. they desire that each passenger’s walking distance $d \leq d_{\text{max}}$. In the case of urban transport, the value $d_{\text{max}} = 0,33$ km is postulated very often.

These two requirements may get into contradiction as can be shown when the network density is discussed.
2.1 Parallel Streets with Urban Transport

Let us look at the situation sketched in Figure 1. We can see a residential district of total width \( a \) km covered by a network of equidistant parallel streets served by public transport. The distance between neighbouring streets is \( x \) km. Hence the total number of streets is \( a/x \). We suppose that the population density is constant in the whole district and that walking speed of a pedestrian here is \( v \) km/h. We expect that pedestrians can walk along the streets and vertically to that direction. The distance between neighbouring public transport stops is supposed equal in each street.

Further we suppose that public transport uses homogeneous rolling stock consisting of equal buses and that there exist \( m \) services to the city centre an hour in total, \( i.e. \) in average \( m/(a/x) = mx/a \) services an hour on each street. Consequently, the average headway is \( a/mx \) hours, \( i.e. \) 60\( a/mx \) minutes.

Choice of the parameter \( x \) influences on two parts of the average passenger time loss:

- average duration of the “vertical” walking to the nearest street \( 0,5(x/2v) = x/4v \),
- average duration of waiting for the next service \( 0,5(a/mx) = a/2mx \).

The length and the duration of walking along the streets is independent from the choice of \( x \) since the distance of stops remains unchanged.
2.2 Optimal Distance of Parallel Streets

In Černý and Kluvánek, (1991, Section 3.4.3) it is proved that the optimal value of the distance $x$ minimizing the time loss of passengers is

$$x_{\text{opt}} = \sqrt{\frac{2av}{m}} \quad (2.1)$$

One has to admit that this value is only theoretical. Usually, it is unfeasible (in practice) due to the following obstacles:

- $a/x_{\text{opt}}$ is not integer and therefore the number of streets $n$ can be, at the best, one of the values: $n = \text{int}(a/x_{\text{opt}})$ or $n = 1 + \text{int}(a/x_{\text{opt}})$.
- If $n$ is the finally chosen number of streets and $m/n$ is not integer, then there are problems with schedule construction.
- Especially in Czech situation, no public transport optimization aspect was usually applied when the district was built up. Transport engineers are asked to cover the district by public transport services only when the set of streets is done already.

Therefore, comparing the factual value $x$ with $x_{\text{opt}}$ serves only as an indicator of the difference between ideal situation and reality. More precisely, ideal with respect to passenger time loss minimization.

Looking at the public transport systems in several Czech towns with population between 40 and 100 thousands, one can observe that if there is a pair or a triplet of more or less parallel streets with urban transport balanced services, then their distance $x$ differs from the value (2.1). Let us see some examples in Table 1. There “pop.” means the total population of the town in thousands, $x$ and $d_{\text{max}}$ mean factual values in the quarter, $d_{th}$ means the theoretical value of $d_{\text{max}}$ if the distance of neighbouring streets is $x_{\text{opt}}$. The average walking speed $v$ is considered the same in all cases, namely $v = 4$ km/h.

Table 1

<table>
<thead>
<tr>
<th>Town</th>
<th>pop.</th>
<th>Quarter</th>
<th>$a$</th>
<th>$n$</th>
<th>$M$</th>
<th>$x$</th>
<th>$x_{\text{opt}}$</th>
<th>$d_{\text{max}}$</th>
<th>$d_{th}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Česká Lípa</td>
<td>40</td>
<td>Slovanka</td>
<td>0.7</td>
<td>2</td>
<td>7.7</td>
<td>0.36</td>
<td>0.85</td>
<td>0.48</td>
<td>0.72</td>
</tr>
<tr>
<td>Česká Lípa</td>
<td>40</td>
<td>Špičák</td>
<td>0.6</td>
<td>2</td>
<td>8.7</td>
<td>0.30</td>
<td>0.74</td>
<td>0.38</td>
<td>0.60</td>
</tr>
<tr>
<td>Děčín</td>
<td>52</td>
<td>Chrást + Letná</td>
<td>1.2</td>
<td>2</td>
<td>4.3</td>
<td>0.60</td>
<td>1.49</td>
<td>0.54</td>
<td>0.98</td>
</tr>
<tr>
<td>Pardubice</td>
<td>91</td>
<td>Dukla</td>
<td>1.4</td>
<td>3</td>
<td>17</td>
<td>0.36</td>
<td>0.81</td>
<td>0.41</td>
<td>0.63</td>
</tr>
<tr>
<td>České Budějovice</td>
<td>96</td>
<td>Šumava</td>
<td>1.2</td>
<td>2</td>
<td>20</td>
<td>0.60</td>
<td>0.69</td>
<td>0.51</td>
<td>0.55</td>
</tr>
<tr>
<td>Hradec Králové</td>
<td>96</td>
<td>Moravské Předměstí</td>
<td>1.2</td>
<td>2</td>
<td>28</td>
<td>0.50</td>
<td>0.59</td>
<td>0.48</td>
<td>0.52</td>
</tr>
<tr>
<td>Ústí nad Labem</td>
<td>99</td>
<td>Severní Terasa</td>
<td>1.2</td>
<td>3</td>
<td>20</td>
<td>0.33</td>
<td>0.69</td>
<td>0.39</td>
<td>0.58</td>
</tr>
</tbody>
</table>
The data, presented in Table 1, were excerpted from the official documents of city administrations and transport companies. The author has to emphasize that she has no evidence about any case of parallel streets with public transport balanced services in middle size Czech towns, where the figures would be markedly different from the ones presented in Table 1. Therefore, one can say that Table 1 may be considered a typical representation of Czech middle size towns.

2.3 Discussion

First thing one can see in Table 1 is the difference between \( nx \) and \( a \) in the half of the cases. It is due to the fact that the district has not been built up in the form \( x/2-x-\ldots-x-x/2 \).

The second is the paradox that in smaller towns \( x_{opt} > a \). The interpretation is that, with respect to time loss minimization, the optimal choice is \( n = 1 \) street with public transport located as close as possible to the middle of the district.

Looking at the data in Table 1 one can see that, save two cases, the optimal values \( x_{opt} \) are approximately twice as much as factual values \( x \). On the other hand, the “ideal” accessibility \( d_{max} = 0.33 \) is reached in no times and only three times \( d_{max}/0.33 \leq 1.25 \). These facts indicate a compromise between transport engineers asking for \( x_{opt} \) and city authorities postulating the value \( d_{max} = 0.33 \), \( i.e. \) 5 minutes of walk. The compromise was evidently based only on intuitive consideration as one can see from great variability of the results. Hence, a more exact method of time and space accessibility harmonization would be welcome. We shall turn to this idea in 2.4.

Moreover, we must not neglect economic aspect of the problem. We have to realize that smaller \( d_{max} \) implies greater number of streets with greater necessity to build and to maintain more shelters at the stops and consequently to greater costs, leading either to greater fares or to greater subsidies. The first possible consequence influences the economic accessibility of public transport for passengers (\( i.e. \) the social pillar), the second one influences the economic pillar of public transport.

2.4 Combined Mathematical Model

In order to formulate a combined mathematical model encompassing time, space and economic accessibility, it is necessary to converse natural values into financial ones. We define the following parameters:

\[ c_1 = \text{(generalized) cost of 1 hour of time loss} \]
\[ c_2 = \text{(generalized) cost of 1 km of walking} \]
\[ c_3 = \text{(genuine) average annual costs of building and maintenance of shelters in one street with public transport in the district displayed in Figure 1, divided by the total annual number of passengers leaving the district or arriving there. One can understand the substance of this cost realizing the fact that if the municipality spends the financial amount } c_3 \text{ on shelter building (directly, or indirectly by subsidies to transport company), it is not able to spend it for another benefit of the inhabitant. Other values mentioned in 2.1 remain unchanged.} \]
2.4.1 Lemma

Let
\[ c(x) = c_1 \left( \frac{x}{4v} + \frac{a}{2mx} \right) + c_2 \frac{x}{4} + c_3 \frac{a}{x} \]  \hspace{1cm} (2.2)

and
\[ x_{opt} = \sqrt{\frac{2av(c_1 + 2c_3m)}{m(c_1 + c_2v)}} \]  \hspace{1cm} (2.3)

then
\[ c(x_{opt}) = \min \{ c(x) : x \in (0; \infty) \} \]

Proof. We have
\[ c(x) = \frac{x}{4} \left( \frac{c_1}{v} + c_2 \right) + \frac{a}{x} \left( \frac{c_1}{2m} + c_3 \right) \]
and
\[ c'(x) = \frac{1}{4} \left( \frac{c_1}{v} + c_2 \right) - \frac{a}{x^2} \left( \frac{c_1}{2m} + c_3 \right) \]
\[ c''(x) = 2 \frac{a}{x^3} \left( \frac{c_1}{2m} + c_3 \right) \]

Since \( c''(x) > 0 \) for each \( x \in (0; \infty) \), the function \( c(x) \) is convex on the whole interval \( (0; \infty) \). Therefore it cannot have more than one stationary point there and such a point ought to be the global minimum on \( (0; \infty) \). It is easy to see that \( c'(x_{opt}) = 0 \). That concludes the proof.

2.4.2 Interpretation

We deal with two types of passengers: \( P_1 \) having the starting point somewhere in the district and the destination in the city centre, \( P_2 \) having their trip in reverse direction.

\( a/x \) represents the number of streets (see Remark 2.2.).
\( x/4 \) represents the (average) walking distance of both \( P_1 \) and \( P_2 \) between the street and the origin/destination in the district.
\( x/4v \) represents the walking time of both \( P_1 \) and \( P_2 \) between the street and the origin/destination in the district.
\( a/2mx \) represents the waiting time of \( P_1 \) at the stop in the district and the same for \( P_2 \) in the city centre.
\( c_1(x/4v + a/2mx) \) represents the lost time “cost” of one passenger.
\( c_2x/4 \) represents the walking “cost” of one passenger.
\( c_3a/x \) represents the portion of shelter building costs of one passenger.
$c(x)$ represents the generalized costs of one passenger.

$x_{opt}$ represents the value of $x$ minimizing the costs $c(x)$.

### 2.5 Discussion

If $c_2 = c_3 = 0$ then the formula (2.3) turns to (2.1). If $c_2 > 0$, it may happen that the municipality decides to neglect $c_3$, i.e. to put $c_3 = 0$ since the factual values are small, e.g. because the shelters are of a simple and cheap type or they are not built at all.

Then we can write

$$x_{opt} = \sqrt{\frac{2av(c_1 + 2c_2m)}{m(c_1 + c_2v)}} = \sqrt{\frac{2av}{m}} \sqrt{\frac{c_1}{c_1 + c_2v}} = \sqrt{\frac{2av}{m}} \sqrt{\frac{1}{1 + \frac{c_2}{c_1}}} = \sqrt{\frac{2av}{m}} \sqrt{\frac{1}{1 + \nu \rho}} = \mu(\rho) \sqrt{\frac{2av}{m}}$$

where $\rho = c_2/c_1$. When we choose, as usual, $\nu = 4$, then the value of the multiplier $\mu(\rho) \in (0; 1)$. For example, if $\rho = 2$, then $\mu(2) = 1/3 \doteq 0.33$, similarly $\mu(1) = 0.45$, $\mu(0.5) = 0.58$, $\mu(0.25) = 0.71$, $\mu(0.1) = 0.85$.

Mainly, the value $\mu(0.25)$ can be considered quite natural since it corresponds to the fact that $4c_2 = c_1$, i.e. for any time $t$ and the corresponding walking distance $4t$ we have $c_1t = 4c_2t = c_22t$. Hence generalized costs of waiting $t$ hours (e.g. sitting in the shelter) is $c_1t$, while walking $t$ hours “costs” $4c_2t + c_1t = c_1t + c_1t = 2c_1t$. In the other words, waiting is two times cheaper than walking.

If the multiplier $\mu(0.25) = 0.71$ is applied to the data from Table 1, i.e. if the values $d_{ih}$ are replaced by $0.71d_{ih}$, these “harmonized” figures become closer to the ones of $d_{opt}$. Of course, we have to emphasize that this is true when the values $c_3$ are neglected. Otherwise, they increase the values of $\mu(\rho)$ back, closer to 1.

### 3. Harmonization within the Economic Pillar

Main Czech public transport systems, i.e. rail, bus and urban ones, are not profitable. All need subsidies from public authorities. We shall deal with three important parameters - costs $c$, revenues $r$ and subsidies $s$ and in Czech situation $c > r$. We do not regard profit of the transport company, since the so called “adequate profit” is a delicate political question now.

The major part of public transport services are ordered by public authorities and they are obliged (by law) to cover the difference $c - r$ by the subsidy $s$, i.e. $s \geq c - r$.

Internal harmonization within the economic pillar does not touch the social one and therefore the level of service must not be changed. Consequently, the parameters $c, r, s$ ought to be harmonized without any impact on timetables.

In order to fortify the economic pillar of public transport sustainability the main goal of this harmonization is to minimize $s$, i.e. to increase $r$ and to decrease $c$. 
3.1 Ablating Spiral: Danger for Revenue

The reduction in subsidies can be reached by several ways. One of them is apparently very appealing, namely a big jump up of fares. The responsible persons believe that it will yield the same revenue jump up. Unfortunately, it is a crucial miscalculation. Instead of revenue increase they can cause an irreversible reduction of demand.

Let us suppose that the public authority together with the carrier decide to increase public transport fares by 20% expecting equal growth of revenue. However, say, 10% of passengers do not accept new prices, abandon public transport and take the natural substitute – individual cars. Hence the revenue increase by 8% only. The fares are increased again, further passengers abandon public transport and the spiral is running, ablating the passenger demand.

3.2 Revenue Maximization via Right Choice of Fares

The facts, mentioned in 3.1 cannot be interpreted as a proof that it is not possible to increase public transport revenue by means of fares.

For the sake of simplicity, let us suppose that in some town there exists only one possibility of payment for the use of urban transport – to buy one way ticket in the price of \( p \) units (e.g. $, £ etc.). The average revenue \( r \) depends on the price \( p \), i.e. \( r = f(p) \). Naturally, one can suppose that \( f \) is continuous. Moreover, \( f(0) = 0 \) and there exists an unacceptable price \( p_o \) giving \( f(p_o) = 0 \) again, since nobody would travel and pay. On the other hand, between the extremes 0 and \( p_o \) there are acceptable prices \( p \) giving positive revenues \( f(p) \). The graph of the function \( f \) may look as the one in Figure 2.

Due to Weierstrass theorem, the continuous function \( f \) on the closed interval \((0; p_o)\) ought to reach its maximal value \( r_{max} \) in an internal value \( p_{max} \in (0; p_o) \). It is quite natural to expect that the function \( f \) is unimodal. And it is reasonable to choose the ticket price exactly \( p_{max} \).

Figure 2

Functional Dependence of Revenue on Ticket Price

\[
r = f(p)
\]

\[
r_{max}
\]

\[
p_{max}
\]

\[
p_o
\]
However, it is not easy to find factual function $f$ and the optimal choice $p_{\text{max}}$ in practice. But, knowing that $p_{\text{max}}$ exists, it needs to make several experiments with different values of $p$ and to deduce how the function $f$ looks like and what could be the value of $p_{\text{max}}$.

### 3.3 Harmonization of Monthly Pass and One-Way Ticket Prices

Let us suppose that in some town there exist two types of public transport tickets:

- one-way ticket for one simple trip in the price $p_1$,
- monthly pass for unlimited number of trips within a month in the price $p_m$.

The author has observed many different values of the ratio $p_m/p_1$ between 12 (Linköping, Sweden, many years ago) and 55 (Prague, several years ago).

Mathematically, the situation is similar as in 3.2., the complication is in the fact that now we deal with the function $f$ of two variables $r = f(p_1, p_m)$ and that the optimum is in a pair $p_{\text{opt}}, \ p_{m\text{opt}}$ which can be approximately found by a series of experiments as well. For the public, the experiments can be substantiated as an attempt to learn the preferences of passengers.

### 3.4 Harmonization within Integrated Transport Systems (ITS)

Harmonization issue is important for integrated transport systems as well. In Czech Republic, one can find ITS in several regions, usually situated around a large city up to a distance of several tens of kilometres (e.g. near Prague, Brno, Ostrava, Olomouc, Zlín etc.). A characteristic feature of each of these areas is that it is operated by several independent transport companies, but passengers can use the services of any of them on an unified ticket. The main problem here is usually considered a harmonious distribution of traffic volumes and revenues among companies. Farsi et al. (2007) described an interesting methodological apparatus for this purpose.

### 4. Harmonization between the Economic and Social Pillars

The term “harmonization” gives the impression that some equilibrium between these pillars is reached. In Czech situation, it is a bit different. Usually, there is a given amount for subsidies and the goal is to maximize the social benefit from public transport. It is not far from the attitude of Hensher (2007) who recommends that the value of money must be considered. The difference is that he calls for cost minimization keeping the desired level of service and we maximize the level of service keeping the acceptable level of subsidies.

There are several hopeful ways to meet the maximization of the social benefits from public transport.
4.1 Optimization of Rolling Stock Structure

“Rolling stock structure” means several groups of mutually interchangeable vehicles, \textit{i.e.} elements of one group ought to have the same usage and more or less the same size. Moreover, the structure is fully determined if the list of groups and numbers of elements of the groups are given.

First, let us look at bus transport, regional and urban. Usually, the following groups are distinguished:

- microbuses with capacity under 15 passengers (usually sitting only)
- minibuses with capacity 15-30 (sitting and standing)
- midi-buses with capacity 31-60 (sitting and standing)
- standard buses with capacity 61-110 (sitting and standing)
- big buses (usually articulated) with capacity over 110 (sitting and standing).

As concerns urban bus transport, no global Czech data are available. For the regional one, the last global data the author has seen are from the year 2000. The figures are: 0\% microbuses, 1\% minibuses, 16\% midi-buses, 79\% standard buses and 4\% big buses. Hence the mean value of bus capacity is about 75.

Newer data are not in such a detailed form, the statistics by Ministry of Transport give only the total number of regional buses 6,110 in this year, almost no change comparing with 6,083 in 2000. Several big providers of regional transport service answered the question of the author that no significant changes have been done during last ten years.

As concerns the passenger demand, the state is similar. More precisely, the demand for regional transport service stagnated or slightly decreased during the last decade. Hence the mean occupancy of 19 passengers in a bus, mentioned in Černá and Černý (2004) has not increased indeed and therefore the disproportion between this figure and 4 times greater mean capacity persists until now. And this is not appropriate ratio probably. But what is the optimal one?

Since more than 85\% of vehicle-kilometres and passenger kilometres are done on working days in Czech regional bus transport, we can discuss mainly that period.

A typical Czech regional bus serves on a line connecting a village V1 with a town T via villages V2, V3, \textit{etc.} with the one way running time under under 30 minutes. It spends the nights in V1. In the morning, it carries out four services from V1 to T with the arrivals about 5:30 (\textit{e.g.} for factory workers), 6:30 (\textit{e.g.} for clerks or shop employees), 7:30 (\textit{e.g.} for pupils or teachers) and 9:30 (\textit{e.g.} for visit to physicians or shopping). Between all such successive “demanded” services there are “return” services, used by a few passengers (\textit{e.g.} teachers or shop employees in villages).

Usually, occupancy of the vehicle during one “demanded” trip V1-T can be approximately expressed in Figure 3.
For \( i = 1, 2, \ldots \) the number \( n_i \) means the increase of passengers in the vehicle in the \( i \)th village, i.e. the number of boarding minus the number of alighting there, whereas \( n \) is the number of alighting in the town. It is obvious that the mean occupancy in this case is \((n_1 + n)/2\).

Figure 3
**Occupancy of the Bus during One “Demanded” Trip**

A direct reflection of Figure 3 can be used for the return trip T-V1 – see Figure 4. There \( m \) is the number of boarding in the town whereas \( m_i \) means the number of alighting minus the number of boarding in the \( i \)th village. The mean occupancy in this case is \((m_1 + m)/2\).

Figure 4
**Occupancy of the Bus during Return Trip**

Let us suppose, for instance, that the vehicle is a standard bus with capacity 70, \( n_1 = 20 \), \( n = 42 \), \( m = 10 \) and \( m_1 = 4 \). Then the mean occupancy of both trips is 19, i.e. 27\%, much more than 19\%. Despite to it, the standard bus can be replaced by a midi-bus with capacity 45, whose operating costs are about 20\% lower.

However, some part (maybe 40\%) of regional buses does not have so simple daily duties as mentioned above. For instance, it happens that the running time from V1 to T is more than 30 minutes and therefore the buses change routes in order to get optimal schedule covering as many trips (services) as possible. Hence the optimization of rolling stock becomes a part of optimal scheduling as described in 4.3.
4.2 Optimal Routing and Frequencing

Mainly in urban transport, one can see that on a route the vehicles (buses, trolley-buses, trams etc.) move from one terminal to the other one and back with regular headways. Then the complete service can be defined by the route and the frequency of services on the route.

Many different approaches are in use in optimization of routes and frequencies. One of the most successful is the one originated by Erlander and Schéele (1974). The optimization procedure starts with a “wide” set \( R_p \) of possible routes, created e.g. by a transport engineer. Then for each \( r \in R_p \) an integer variable \( x_r \) is defined, expressing the number of buses (or trams etc.) assigned to the route \( r \). Optimal values of these variables are found by means of mathematical programming.

The original Erlander and Schéele (1974) approach uses the passenger time loss as the objective function. However, the waiting time on the route \( r \) is proportional to the headway, i.e. to \( 1/x_r \), and therefore the problem became non-linear and difficult for solution.

In order to overcome this inconveniency, Černá and Černý (2004, section 12.3.1) chose a different objective function, namely the size of space for the passenger, which is proportional to \( x_r \). The problem became linear and enabled to use standard LP solvers.

As concerns the pure frequencing in the case when the route has been already done, Van Reeven (2008) showed that under some assumptions, considering private car as the main alternative to public transport, a monopolist profit maximizing frequency is at least as high as the welfare-maximising frequency (= the frequency minimizing passenger time loss).

4.3 Optimization of Rolling Stock and Crew Scheduling

In this context, scheduling means creation of sequences

\[
S_1 = e_1(1), ..., e_i(q_i) \\
.................................. \\
S_n = e_n(1), ..., e_n(q_n)
\]

(4.1)

where \( e_i(j) \in E, i = 1, ..., n, j = 1, ..., q_i \) and \( E \) is a set of elementary paces of work of vehicles or crew members (usually drivers). It represents an “atom” from the view-point of scheduling. It is not allowed to be divided into smaller parts.

A sequence \( S_i \) represents usually one daily duty for a vehicle (in the case of vehicle scheduling) or for a person, e.g. a driver (driver scheduling).

As concerns scheduling of bus drivers, there are some Czech particularities, not to be seen in the major part of other countries. The main one is that a Czech driver does
not change buses. Even during a long period. A driver is unambiguously assigned to a concrete vehicle and a change occurs only when the vehicle is unable to work. Therefore one can distinguish only two types of bus daily duty:

- one-driver bus duty \( S_i = e_i(1), \ldots, e_i(q_i) \)
- two-drivers bus duty \( S_i = e_i(1), \ldots, e_i(q_i^1), e_i(q_i^1 + 1), \ldots, e_i(q_i^2) \), consisting of two driver duties \( e_i(1), \ldots, e_i(q_i^1) \) and \( e_i(q_i^1 + 1), \ldots, e_i(q_i^2) \).

This is a disadvantage deteriorating the effectiveness level of a vehicle, since the mandatory pause of a driver (e.g., 40 minutes after 4 and half hours of driving) causes the equal pause of the vehicle. On the other hand, it is an advantage since it is enough to create one set of daily duties, fulfilling the requirements both for vehicles and drivers.

Duration of one daily duty \( S_i = e_i(1), \ldots, e_i(q_i) \) starts some “manipulation time” \( t_b \) before the starting (departure) time of \( e_i(1) \) and ends the time \( t_e \) after the end (arrival) of \( e_i(q_i) \). Since it represents the duration of work of one or two drivers, several values of it are more preferable than the others. For “one-driver duty”, the acceptable duration is 6-9 hours, for “two-driver duty”, it is 12-18 hours. Therefore some duty duration penalty is included into optimization of scheduling problem.

Another parameter to be taken into account is the least necessary capacity \( k(e_i(j)) \) of the bus assigned to the service \( e_i(j) \). Then the least necessary capacity \( k(S_i) \) of the duty \( S_i \) is

\[
k(S_i) = \max\{k(e_i(1)), \ldots, k(e_i(q_i))\}
\]

Let us suppose that the company possess the rolling stock \( B = \{b_1, \ldots, b_r\} \) consisting of buses with capacities \( k(b_1), \ldots, k(b_r) \). In Czech practice, the number of available buses \( r \) of the company is much greater than the maximal number \( n \) of daily duties. The greatest \( n \) is usually reached on Mondays during the school year. Usually it is \( 1.2n \leq r \leq 2n \). Moreover, it does not happen that a company has a lack of greater vehicles, i.e., it does not happen that a bus cannot take all waiting passengers. The contrary uses to be true. A bus \( b_i \) is assigned to a duty \( S_i \) and \( k(b_i) \) is much greater than \( k(S_i) \). It may also happen that for each \( i \) the difference between \( k(b_i) \) and \( k(S_i) \) is zero or a small positive number, but after an optimization of daily duties the differences become much greater. No motivation for such an optimization may be a consequence of the bad rolling stock structure, as mentioned in 4.1.

The economic consequence of such disproportion is evident. Greater bus has greater cost of the same elementary piece of work. The author’s experience shows that, roughly saying, \( m \) times greater bus capacity implies (approximately) \( \sqrt{m} \) times greater cost of the same work.

Although there are several optimization techniques and many commercial software products for bus and driver scheduling available in the world, the author has not met any Czech bus scheduling made by other methodological principle than “KASTOR”, created and innovated by S. Palúch (1998). It works in two steps:

1st: Minimization of the number \( n \) of daily duties, i.e., the number of necessary vehicles. This step is done by an exact method based on graph theory.
2nd: For the given \( n \), minimization of a complex objective function, containing several components, expressing e.g. the requirement of

- closeness of the daily duty \( S_i = e_i(1), \ldots, e_i(q_i) \), (the initial terminal of \( e_i(1) \) ought to be equal to the final terminal of \( e_i(q_i) \)),
- as small capacities (and generated costs) of assigned vehicles as possible,
- accordance with the rules on drivers work pauses,
- accordance with acceptable duration of the duty,
- and others, if necessary.

The 2nd step is done by a crossing heuristics.

### 4.4 Right Ways to Minimization of Costs and Positive Social Impact

The author headed regional transport optimization project in the district of Tachov (Western Bohemia) several years ago. The result was that the same time table, i.e. the same set of elementary services \( E \), was divided into new set of daily duties, saving about 10\% of costs comparing with the former solution, which represented about 20\% of subsidies. The main sources of savings were first in the minimization of the number of daily duties (and the assigned buses) and second in the decrease of bus capacities. Then the regional authority asked the research team to increase the set \( E \) and to spend the saved money in favour of the accessibility of transport, i.e. in favour of the social pillar. Upon that several new bus stops and new routes covering new micro-regions were created and introduced into service.

This example demonstrates how the bus and crew scheduling can help in harmonization between the economic and social pillars. Decreasing the costs it can strengthen the economic pillar or using the saved money it can consolidate the social one. Or, it is possible to do something in between. One can say that **optimization of vehicle scheduling and optimization of rolling stock structure** are two basic ways to harmonic fastening of economic and social pillars in the case of regional transport.

This thesis has to be modified for urban transport. The reason is based on the fact that, with few exceptions, the vehicle daily duties have a very simple setup: From the beginning to the end to move on the same route from one terminal to the other and back. It is not a field of optimization at all. Another source of savings can be demonstrated by an “educational” example in Figure 5.

One can see a diametric route \( A-B-C-D \) from one suburb to another via centre of town there. \( A-B \) and \( C-D \) are suburban parts, \( B-C \) is in the centre of town. The running times including stopping are 19, 17 and 19 minutes. The demanded capacity in one direction is 520 an hour and 500 an hour in the suburbs, 1040 an hour in the centre.
Let us suppose that there passes one urban bus route No 1, having assigned 20 buses, each with capacity 90 passengers (sitting and standing). The complete vehicle return time is 120 minutes, consisting of $2(19 + 17 + 19) = 110$ minutes of running and $5 + 5$ minutes dwelling in the terminals $A$ and $D$. The supply represents 10 services an hour in one direction, i.e. 900 places for passengers. This represents surplus 380 and 400 places in the suburbs respectively, but there is a lack of 140 places an hour in the centre. Buses are overcrowded there.

And now, let an optimization is applied. The route 1 is shortened to $A-B-C$ and a new route 2 passing $B-C-D$ is introduced. Both routes are assigned by 8 buses. They have vehicle return times 80 minutes, consisting of $2(19 + 17) = 72$ minutes of running and $4 + 4$ minutes of dwelling. Both supply 6 services an hour in one direction, i.e. 540 places for passengers. That represents 1080 places on the common leg $B-C$ in the centre. The supply is greater than the demand anywhere and the total number of assigned buses is 16 only. The costs of 4 buses are saved.

The author’s experience shows that optimization of routing and frequenting in urban transport may save 5-10% of costs when the size of buses remains unchanged. Introduction of smaller buses may increase the saving a little, but this gain is much smaller than in the case of regional transport.

Similarly as the optimal scheduling in regional public transport, routing and frequenting in urban transport can be optimized by various methods and software products. In Czech and Slovak Republics many times, maybe in major part of projects, the PRIVOL method, described by Černá and Černý (2004, Section 12.3.1) is used.

5. Conclusion

The paper has shown that the three pillars of sustainability have specific position in the case of public transport, namely after the recent financial crisis. The message for public authorities can be divided into several parts.

First of all it is emphasized that the base of social pillar is situated in the accessibility of transport service. It is important to reach a balance of time, spatial and economic requirements.

Afterwards a particular situation in seven selected middle size Czech towns is analysed and compared with optimal parameters derived from theoretical models. The public authorities are motivated to analyse the results and to adopt appropriate actions. The author proposed her own results for this purpose.
Hereafter, internal harmonization within the economic pillar was introduced as another important issue. Right choice of fares and right balance between the prices of one-way ticket and monthly pass was emphasized.

Apart from internal harmonization inside pillars the external one could not be ignored. It came to the turn next. The crucial role of relation between the economical and social pillars was accented. There were presented several measures leading to smaller costs without any detriment of accessibility as the fundament of the social pillar. The main ones were optimization of rolling stock structure, routing, frequenting and scheduling. Of course, the crew scheduling was not forgotten either. However, the opposite question concerning the increase of social benefits keeping the costs unchanged was risen and answered as well.

The author is convinced that there exist right ways to take and false ones to identify and avoid. She hopes that the paper has helped in that.

Nevertheless, certainly it is not possible to say that no other measures can be developed and implemented to practice. For instance in the revenue maximization as a counterbalance to cost minimization would be worth a try.

References


